Magnetohydrodynamic Turbulence in Accretion Disks

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ABSTRACT

Recent work on the structure of magnetic fields in a turbulent medium gives predictions for the properties of the magnetic flux tubes as a function of the Mach number and scale of the turbulence, and the resistivity and viscosity of the fluid. Here I discuss the implications of this work for accretion disks. I show that although accretion disk flux tubes are usually almost completely evacuated, they are nevertheless less buoyant than previous estimates have suggested. I also note that vertical magnetic flux tends to be ejected from the outer edge of accretion disks, so evidence for continued magnetic activity in such systems should be interpreted as supporting the existence of dynamo activity.

Subject headings: **ACCRETION, ACCRETION DISKS - PLASMA**

1. INTRODUCTION

Turbulence and magnetic fields are both topics of morbid curiousity in astrophysics. In that context they are usually seen as poorly understood, undoubtedly real phenomena that can be used as part of an explanation of last resort, i.e. when all calculable models have been disproven. Consequently both these phenomena, together and separately, have been used in constructing models of angular momentum transport in accretion disks, another process of indisputable reality whose nature is obscure. Notwithstanding this troubled history, I will present in this paper a summary of my recent work on magnetic fields in turbulent media and explore its implications for accretion disks.

I have three basic reasons for pressing ahead with such an unpromising topic. First, the structure of magnetic fields in accretion disks determines the rate at which magnetic flux is lost from the disk. This implies that one can get a variety of rates depending on one's model for the magnetic field structure, but it also implies that any physically well

motivated model can have interesting, and possibly unique, implications. Second, magnetic field instabilities represent a mechanism guaranteed to move angular momentum outward and matter inward (Balbus & Hawley 1991), which is not true for many of the instabilities suggested as the basis for dissipation in accretion disks. It is therefore critical to explore the nature of the turbulence resulting from these instabilities. Third, vertical magnetic fields, entrained in accretion disks, are widely believed to be responsible for driving violent outflows, especially jets, from a wide variety of accretion disks. The radial transport of vertical magnetic flux can't be understood without examining the nature of magnetic fields in turbulent disks, and the global structure of accretion disks fields hinges on this issue.

Before I begin I need to summarize the relevant features of accretion disks. The single most important point is that they are luminous due to the conversion of orbital energy into heat. This implies an outward flux of angular momentum. Such a flux would follow from the existence of some local viscosity, but it would have to be much larger than the viscosity implied by microscopic processes. The usual solution is to invoke an effective viscosity due to collective processes which is $\nu \equiv \alpha h c_s$, where h is the disk height, c_s is the sound speed, and α is an arbitrary constant of order unity (Shakura and Sunyaev 1973). For a thin disk with no self-gravity, i.e. a 'Keplerian disk', we have a rotational frequency $\Omega(r) \propto r^{-3/2}$, a disk height $h \sim c_s/\Omega$, and, by definition, $h \ll r$. In this case the differential rotation in the disk plus the assumed effective viscosity leads to an inward flux of mass given by

$$\dot{M} \sim \alpha \Sigma h^2 \Omega, \tag{1}$$

and a radiative flux from the disk surface of

$$F_{radiative} \sim \dot{M} \Omega^2. \tag{2}$$

Attempts to model the outbursts for dwarf novae and X-ray transients have led to the conclusion that α is probably not a constant, but a function of local conditions (Cannizzo 1994 and references therein). Assuming that α goes as $(h/r)^n$, where n is a constant of order unity, gives a reasonable fit to the observations.

In §II of this paper I will summarize my recent work on the distribution of magnetic fields in a turbulent medium. In §III I will discuss the implications of this work for magnetic buoyancy in disks and how this leads to a direction connection between dynamo growth rates in disks and the appropriate value of α. In §IV I will draw some general conclusions and point the way towards future progress on this topic.

2. MAGNETIC FIELDS IN TURBULENT FLUIDS

The material in this section is a synopsis of Vishniac (1995a). The basic feature of this model is that the magnetic field in a high β fluid, i.e. one in which the magnetic field pressure is small compared to other sources of pressure, is spatially intermittent. Most of the magnetic flux is contained in flux tubes, whose radii are much smaller than the scale of curvature for the field. This is not a novel suggestion. In fact, it is about what one would guess from examining the magnetic field in the photosphere of the Sun. It does raise the question of how such flux tubes form. Why should the magnetic field and the gas spontaneously separate from one another? The mechanism I have proposed is a process I call turbulent pumping. If we consider an isolated flux tube in a turbulent medium, then as long as the flux tube is flexible enough to respond to the hydrodynamic forces exerted by the surrounding fluid then it will undergo stretching at a rate roughly equal to the shearing rate on the scale of curvature of the flux tube. This lowers the linear density of matter in the flux tube at the same rate. By the time the flux tube length has doubled it will be twisted by the surrounding flow in such a way that it will intersect itself, or a whole set of neighboring flux tubes. This will result in formation of a set of closed loops which will shrink down to dissipative scales and vanish, thereby maintain a constant flux tube length in the turbulent fluid. In this way matter is removed from the magnetic flux tubes at a rate which is dependent only on the properties of the turbulent medium and not at all on the specific resistivity of the fluid. In a stationary state this loss of matter from the flux tubes is balanced by ohmic diffusion of the charged particles onto the field lines, a process which becomes extremely slow as the conductivity of the fluid increases to astronomical values. The consequence is the appearance of flux tubes whose internal gas density can be far below that of the surrounding fluid. In the Sun flux tubes will be largely evacuated only near the top of the solar convection zone, whereas in accretion disks flux tubes will be almost empty whenever the disks are largely ionized.

This process of turbulent pumping depends on several conditions. First, there can be little or no turbulent diffusion of matter into the flux tubes. Otherwise the mismatch between the mass loss driven by collective processes (flux tube stretching and the creation of closed loops) and mass loading driven by ohmic diffusion will disappear. Preliminary work indicates that this condition will be satisfied whenever the Alfvén speed in the flux tubes is significantly greater than the turbulent velocity outside. In other words, this condition is satisfied self-consistently when turbulent pumping is effective. The transition from a diffuse field to one contained in flux tubes is not yet understood. Second, reconnection must be efficient, in the sense of allowing the magnetic field to rearrange its topology in less than an eddy turn over time. Once again, this condition is met self-consistently in the flux tube model. This result assumes the Sweet-Parker rate for reconnection, which is generally considered to the slowest reasonable estimate for reconnection rates. Third, turbulent

pumping relies on the notion that closed loops whose radii are less than a typical eddy size will tend to shrink to dissipative scales quickly, thereby unloading their entrained mass into the surrounding plasma. This also follows from the physics implicit in the flux tube description of the magnetic field, although the loops tend to shred as they shrink, thereby creating a more diffuse, but weaker and less organized, component to the magnetic field.

What does this pumping lead to? If the resistivity is large enough that particles can diffuse to the center of a flux tube in less than one eddy turnover time, then the flux tube radius is

$$r_t \approx \left(\frac{\eta}{kV_l}\right)^{1/2}, \tag{3}$$

where η is the resistivity, V_l is the turbulent velocity on the scale l on which the flux tubes are bent, and k is the corresponding wavenumber, $k = 2\pi/l$. In this limit the magnetic pressure in a flux tube has a gaussian profile. When the resistivity is sufficiently small the flux tubes will become largely empty, i.e. the magnetic field inside the flux tube, B_t, is given by

$$\frac{B_t^2}{8\pi} = P, \tag{4}$$

where P is the pressure of the surrounding fluid. In this limit each flux tube will have a skin depth of width $(\eta/kV_l)^{1/2}$ surrounding a hollow core.

None of this tells us to how decide what r_t or l should be in a given situation. For this we need to understand the forces between flux tubes, since these forces will determine the distribution of flux elements in the turbulent fluid. Since in this picture the magnetic flux is confined to the interiors of the flux tubes the forces between them will be hydrodynamic and stem from the turbulent wakes created as the fluid moves past the semi-rigid flux tubes. The most obvious effect (cf. Parker 1979 §8.9) is the attraction between flux tubes when one lies downstream from the other. This is simply due to the fact that the downstream flux tube feels a reduced turbulent drag since the momentum flux around it is reduced by $\rho V_l^2 r_t/w(r)$, where ρ is the fluid density, V_l is the fluid velocity relative to the flux tubes, r_t is the typical flux tube radius, and $w(r)$ is the width of the turbulent wake at a distance r downstream from the leading flux tube. This attraction is analogous to mock gravity, in that it stems from the ability of neighboring tubes to block statistically isotropic repulsive forces in the environment. It is less well known that one expects neighboring flux tubes whose separation is more or less perpendicular to the ambient flow to repel one another. This is known experimentally (Zdravkovich 1977, Gu, Sun, He, & Zhang 1993) since no adequate analytical treatment of the near-field turbulent flow is available. Unlike the shielding effect this repulsion depends critically on the nature of the flux tube wakes. When the wakes are purely laminar and stable there is an attractive force, which is the basis for previous claims that flux tubes embedded in a turbulent flow always attract one another

(Parker 1979 §8.9). The transition to an unstable wake, capable of producing repulsion, occurs when

$$\frac{V_l r_t}{\nu} >\sim \pi^3. \tag{5}$$

In other words, when the Reynolds number *on the scale of the flux tube radius* exceeds a critical value which lies in the range of 30 to 40. When this criterion is not satisfied flux tubes will aggregate. At higher Reynolds numbers the magnetic field will be broadly distributed through the fluid in the form of discrete flux tubes that maintain their separate identities through a rough equilibrium between attractive and repulsive interactions. This qualitative difference is significant, since this criterion is almost always satisfied in astrophysical objects, and not (yet) satisfied in numerical simulations.

Neglecting viscosity, which is reasonable in stars and accretion disks, I have used these points to construct a simple model of the magnetic field structure in a turbulent conducting fluid. A detailed discussion is given in Vishniac (1995a). Here I simply quote the relevant results. First, if the magnetic field energy density is comparable to, or less than, the turbulent energy density, there exists some scale l such that

$$\frac{B_t^2}{l} \sim \rho \frac{V_l^2}{r_t}. \tag{6}$$

The left hand side of this equation is the force per volume in the flux tube exerted by magnetic tension. The right hand side is the force per volume exerted by turbulent drag from the ambient fluid. Their rough equality defines the scale l as the scale of curvature for a typical flux tube. In typical turbulent cascade $V_l^2 l$ is a sharply increasing function of l. Consequently, on scales larger than l the magnetic field is almost passively advected. On smaller scales the flux tubes are almost rigid.

Second, in order to balance the time-averaged attractive and repulsive forces between flux tubes, it is necessary to suppose that on all scales less than l the number of flux tubes within a radius r of a given flux tube, N_r, satisfies the condition

$$N_r r_t \sim r. \tag{7}$$

This defines a fractal distribution of dimension one which extends from the flux tube radius up to the scale l.

I can combine these results to get an expression for the average magnetic field energy density. This average is well-defined only on scales larger than l. On that scale I obtain

$$\langle B^2 \rangle \sim N_l B_t^2 \left(\frac{r_t}{l}\right)^2, \tag{8}$$

or

$$\langle B^2 \rangle \sim B_t^2 \frac{r_t}{l}. \tag{9}$$

Combining this with equation (6) we obtain

$$\langle B^2 \rangle \sim \rho V_l^2. \tag{10}$$

In other words, the scale of curvature for the flux tubes is the scale of equipartition between the mean square magnetic field and the average turbulent energy density.

3. BUOYANCY AND DYNAMOS IN DISKS

Given this specific model for the structure of a magnetic field in a turbulent medium it is possible to discuss the systematic motion of a magnetic field in an accretion disk. I begin by noting that the rate at which a flux tube will rise due to buoyant forces is given by the balance between turbulent drag and the buoyant acceleration. For a flux tube this gives

$$\Delta\rho(\pi r_t^2)g \sim \rho V_b V_l r_t, \tag{11}$$

where $\Delta\rho$ is the density deficit inside the flux tube, g is the local gravitational acceleration, and V_b is the buoyant velocity. The left hand side of this equation is the buoyant force per unit length. The right hand side is the turbulent drag, assuming that $V_b \ll V_l$. In what follows I will assume that the magnetic field is in equipartition with the turbulence and write V_T instead of V_l. Assuming that the flux tube is small enough to be in good thermal contact with the surrounding medium, which is usually reasonable, the fractional density deficit $\Delta\rho/\rho$ is roughly the ratio of the magnetic pressure in the flux tube to the ambient pressure. This implies that

$$\left(\frac{B_t^2}{8\pi P}\right)\rho(\pi r_t^2)g \sim \rho V_b V_l r_t, \tag{12}$$

or

$$V_b \sim \left(\frac{\rho g}{P}\right) L_T V_T \sim \frac{L_T}{l_p} V_T, \tag{13}$$

where l_p is the local pressure scale height and I have used the condition that the radius of curvature of the magnetic field lines is L_T. Note that if the magnetic field energy is below equipartition then I need to replace $L_T V_T$ with the appropriate lV_l, implying a slower buoyant rise. For stellar convective turbulence $l_p \sim L_T$ and magnetic flux will rise at substantial fraction of the local turbulent velocity once the magnetic field reaches equipartition with the turbulence.

This result cannot be extended to accretion disks. There the most plausible source of turbulence is magnetic field instability first described by Velikhov (1959) (see also Chandrasekhar 1961) and applied to accretion disks by Balbus and Hawley (1991). Here I invoke the description of the saturated state of this instability for a large scale azimuthal field embedded in an accretion disk given in Vishniac and Diamond (1992). The dominant eddies will be those characterizing the fastest growing mode, for reasons explained in that paper. The instability will saturate in a turbulent state characterized by a typical turbulent velocity comparable to the Alfvén speed, i.e. $V_T \sim V_A$. The eddy size will be $L_T \sim V_A/\Omega$, where Ω is the local rotational frequency. This gives rise to an effective viscosity and diffusion coefficient of order $L_T V_T \sim V_A^2/\Omega$ or $(V_A/c_s)^2 h^2\Omega$, where h is the disk thickness and I have used the relationship $c_s \sim h\Omega$, which applies to thin, non-self-gravitating accretion disks. The dimensionless viscosity of the disk, due to magnetic field stresses, is just $(V_A/c_s)^2$.

What does this imply about flux tubes in accretion disks? It can be shown (Vishniac 1995b) that flux tubes in hot disks are in the ideal fluid regime, i.e. they are almost completely empty. In this case

$$r_t \sim L_T \left(\frac{V_T}{c_s}\right)^2 \sim \left(\frac{V_A}{c_s}\right)^3 h \sim \alpha^{3/2} h. \tag{14}$$

The buoyant velocity is

$$V_b \sim \frac{L_T}{l_p} V_T \sim \frac{V_A/\Omega}{h} V_A \sim \frac{V_A^2}{c_s} \sim \alpha c_s. \tag{15}$$

Since the loss rate for magnetic flux is just $V_b/h \sim \alpha\Omega$ this implies that the azimuthal magnetic field escapes from the disk at a rate which is comparable to the thermal relaxation rate for an optically thick disk. I note in passing that this flux loss rate is smaller, by a factor of V_A/c_s, than estimates based on the Parker instability, which is normally taken to imply a buoyant velocity $\sim V_A$. The reason for this discrepancy is that the Parker instability is strongly suppressed by the Balbus-Hawley instability (Vishniac and Diamond 1992). In a stationary state the magnetic field must be regenerated by some dynamo process so that the dynamo growth rate balances the buoyant flux losses, i.e.

$$\Gamma_{dynamo} \sim \frac{V_b}{h} \sim \alpha\Omega, \tag{16}$$

or

$$\alpha \sim \frac{\Gamma_{dynamo}}{\Omega}. \tag{17}$$

When radiation pressure is large and electron scattering dominates the opacity, a situation normally encountered in the inner regions of AGN disks, the flux tube properties

change significantly. In this case the magnetic pressure in the flux tubes is limited by the ambient gas pressure, since the ambient photons can diffuse into the flux tubes on a very short time scale. This does not affect the efficiency of turbulent pumping, with the consequence that the flux tubes are larger than one would expect in the ideal fluid regime, but are still evacuated. This gives a modified expression for the flux tube radius, i.e.

$$r_t \sim L_T \left(\frac{V_T}{c_{s,gas}}\right)^2 \sim h \left(\frac{V_A}{c_s}\right)^3 \frac{P}{P_{gas}} \sim \alpha^{3/2} h \frac{P}{P_{gas}}. \tag{18}$$

This leads to an enhanced buoyancy so that

$$V_b \sim g \left(\frac{\rho V_T^2 L_T}{P_{gas}}\right) \frac{1}{V_T} \sim \frac{P}{P_{gas}} \alpha c_s. \tag{19}$$

Consequently, for a given dynamo growth rate, balancing magnetic flux generation with buoyant losses I get

$$\alpha \sim \left(\frac{\Gamma_{dynamo}}{\Omega}\right) \left(\frac{P_{gas}}{P}\right). \tag{20}$$

In other words, as long as the dynamo growth rate is independent of the magnetic energy density the implied disk viscosity will scale with the gas pressure, rather than the total pressure. Of course, this may imply that other angular momentum transport mechanisms, normally dominated by magnetic stresses, become important.

What are some possible disk dynamos?

The equilibrium state of the azimuthal magnetic field is determined by the balance between dynamo activity and vertical buoyancy. However, one can also imagine that a typical accretion disk will have a large scale vertical magnetic field, if for no other reason than the fact that such a field is likely to be accreted along with matter added to the outer edge of the disk. Clearly vertical buoyancy is irrelevant to the evolution of this field. Moreover, since the field lines cross the disk in concentrated flux tubes, and spread out above and below the disk, the tension due to strong bending of the external field lines is negligible. Nevertheless, there are two effects which will tend to move vertical field lines outward. First, if the field lines are bent radial by some total angle 2θ as they cross the disk, then turbulent diffusion through the disk will tend to combine radial field lines of opposite polarity, moving the point at which the field lines cross the disk outward at a rate of roughly $\alpha c_s \tan\theta$ (Van Ballegooijen 1989). If the magnetic field curvature is determined only by large scale stresses than $\theta \sim h/r$ and this velocity is comparable to the inward flow of matter in the disk. Consequently it will be difficult to determine the direction of drift for the magnetic field. Of course, if the concentration of magnetic field in the inner disk increases, then the global stresses will increase and the disk will stop accreting vertical flux

regardless. In addition, if the field is responsible for driving a wind or jet (cf. Shu et al. 1994 and references therein) then it will tend to bend sharply near the disk which will move the field outward.

Second, the flux tubes containing the vertical flux will be subjected to radial buoyancy forces (Parker & Vishniac 1995). Each flux tube will have an associated energy due to its displacement of matter and associated pressure. This energy has two (usually) comparable parts. The first is due to the surrounding pressure and is roughly equal to $\Delta P L\pi r_t^2$, where L is the length of the flux tube, and ΔP pressure contributed by the magnetic field in the flux tube. The other is due to the displacement of matter which could otherwise settle to the disk midplane. This term is of order $\Delta\rho(h\Omega)^2 L\pi r_t^2$, where $\Delta\rho$ is the density deficit in the flux tube and is of order ρ under normal circumstances in a hot disk. For a gas pressure dominated disk the two are comparable and roughly equal to $B_t^2 r_t^2 L$. For a radiation pressure dominated disk the gravitational term dominates is larger by a factor of P/P_{gas}. One would get the same effect by replacing the ΔP in the pressure contribution with P rather than P_{gas}. If the energy associated with a flux tube is U_t then the consequent radial drift velocity is obtained by equating the turbulent drag with the radial gradient of U_t, or

$$\rho V_T V_b r_t \sim -\frac{\partial_r U_t}{L} \sim -P r_t^2 \partial_r(\ln U_t). \tag{21}$$

Consequently,

$$V_b \sim -\alpha c_s h \frac{P}{P_{gas}} \partial_r(\ln U_t), \tag{22}$$

which for $\partial_r(\ln U_t) \sim r^{-1}$ implies a radial drift velocity which is larger than the inward accretion velocity by a factor of P/P_{gas}. The direction of the drift depends on the sign of $\partial_r(\ln U_t)$. Since a single flux tube can break apart, or combine, in the course of its radial drift, we need to evaluate this derivative under the constraint that the magnetic flux remains fixed, or that the area goes as $P_{gas}^{1/2}$. The length of the flux tube will exceed h, since each tube actually crosses the disk in a random walk. We will assume that $L \propto h\alpha^{-1/2}$. Subject to these constraints we find that

$$V_b \sim -\alpha c_s h \frac{P}{P_{gas}} \frac{1}{2} \partial_r \left(\frac{P \dot{M} c_s}{P_{gas}\alpha} \right). \tag{23}$$

If $\dot{M}$ is constant then this will almost give a strong outward buoyancy to the flux tubes, which will clearly dominate over accretion when $P \gg P_{gas}$. In the event that the inner disk is unstable and $\dot{M}$ varies with r there will still be a averaged outward flow. The evidence for magnetic activity in accretion disks, especially in AGN, must be read as evidence for large scale dynamo activity in these disks.

What are the prospects for a reasonable theory of dynamo activity in disks? There is an extensive literature on this topic, too extensive to summarize here. My own view is that there are only a few processes which we can be reasonably sure exist and which may dominate in real accretion disks. All of them rely on the notion that shearing of a radial field provides for efficient generation of a large scale azimuthal field. The tricky step is to understand how the azimuthal field component regenerates the radial field. One of the most popular notions is that magnetic buoyancy produces turbulent motions with a preferred helicity, which close the cycle by twisting the azimuthal field lines into radial field lines (Galeev, Rosner, & Vaiana 1979). However, this relies on using the Parker instability, which has a large azimuthal wavenumber. Since modes with a large k_θ and a slow growth rate will be suppressed by the Balbus-Hawley instability, this mode is not expected to exist in real accretion disks (Vishniac and Diamond 1992), nor is it seen in numerical simulations (Brandenburg et al. 1995). Another idea is that internal waves, excited near the outer edge of the disk via tidal forcing (Goodman 1993), will fill the disk and drive a dynamo with a growth rate $\sim (H/r)^{3/2}$ (Vishniac, Jin, and Diamond 1990, Vishniac and Diamond 1992). This implies a dimensionless viscosity of $\sim (H/r)^{3/2}(P_{gas}/P)$. Finally, Balbus and Hawley (1991) have claimed that the Balbus-Hawley instability will lead to a local turbulent dynamo which will saturate near equipartition between the field and ambient pressure, leading to an α of order unity. Something like this is seen in numerical simulations (e.g. Brandenburg et al. 1995), although it would appear to be inconsistent with phenomenological work on accretion disks (cf. Cannizzo 1994 and references therein). On the other hand, the numerical results are also consistent with the idea that the Balbus-Hawley instability is driving an incoherent dynamo according to mean field theory, leading to a saturation which depends on the geometry of the computational box (Vishniac and Brandenburg 1995). This theory predicts an effective α of order $(H/r)^2(P_{gas}/P)^6$. The steeper scaling with H/r and strong suppression in radiation pressure dominated environments implies that while this process may play a role in real disks, it will only dominate in disks not subject to significant elliptical distortions at large radii and free of significant radiation pressure.

4. CONCLUSIONS AND FUTURE PROSPECTS

I can summarize my results as follows. First, I have proposed a simple model of flux tube formation that is consistent with solar observations. Second, in this picture there is a failure of 'flux-freezing' to adequately describe the macroscopic motions of the field and fluid. In general there will be some significant relative motion between the flux tubes and the fluid. This suggests the possibility of mean-field dynamos. Third, direct numerical

simulation of astrophysically realistic driven MHD turbulence is not currently possible. Turbulence driven by magnetic field instabilities can be simulated to obtain qualitative results, but such simulations will be quantitatively unreliable. Fourth, accretion disks might be able to move magnetic field lines inward, but only if their radial bending angle across the disk is of order h/r or less. Even in this case radial buoyancy may move them outward. Fifth, magnetic viscosity in AGN disks couples primarily to gas pressure, not radiation pressure. This may imply that the viscosity due to hydrodynamic effects, e.g. internal wave breaking, dominates. Finally, given the relatively high rate of vertical and radial buoyant magnetic flux losses, evidence for continued magnetic activity is disks (and stars) can only be explained by the presence of some dynamo mechanism.

It is somewhat disappointing that the MHD turbulence model used here implies that numerical simulations of astrophysical turbulence are not physically realistic. Fortunately, this same theory does predict scaling behavior and approximate saturation values for magnetic fields in the viscous regime, which is currently accessible to numerical simulations. Somewhat fragmentary results from current work seem to show the predicted behavior of the magnetic field as a function of Reynolds number (Vishniac 1995a). Future tests of the theory in this regime should give us some confidence in its application to astrophysics, or allow us to discard it in favor of some alternative description. However, until we are in position to do more realistic MHD simulations it will not be possible to replace the approximate results sketched here with quantitative estimates.

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